4-2 Exponential Equations

4-2a: I can use exponential formulas to model and solve situations of growth and decay.

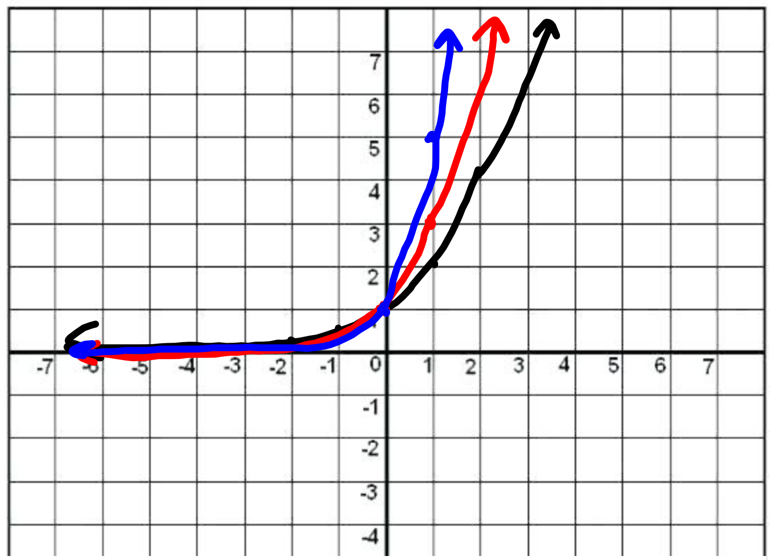
EXPONENTIAL FUNCTION

$$f(x) = a(b)^x$$

- a: Initial Value (y-intercept)
- b: Base (Multiplier)
- x: Exponent

$$f(x) = 2^x$$

x	2^x
-2	0.25
-1	0.5
0	1
1	2
2	4



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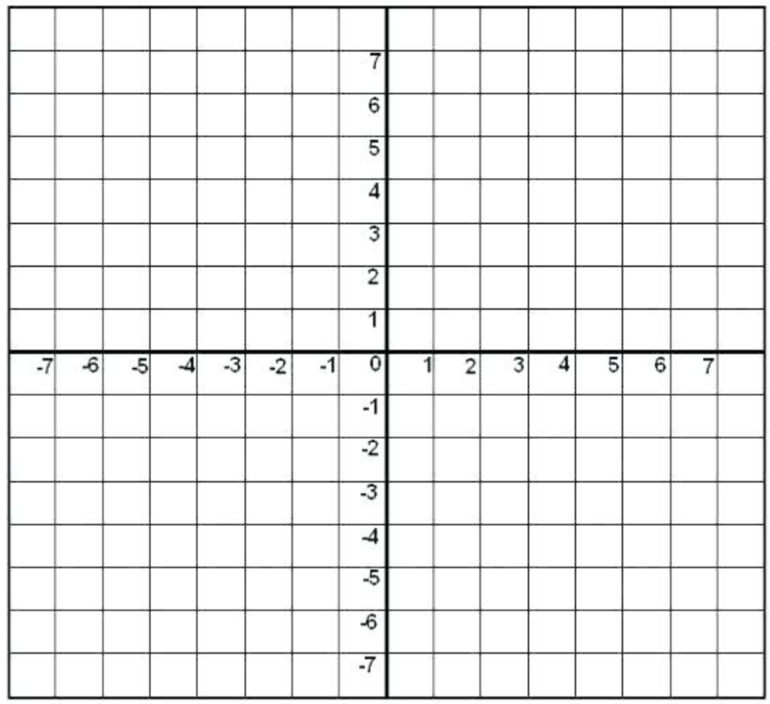
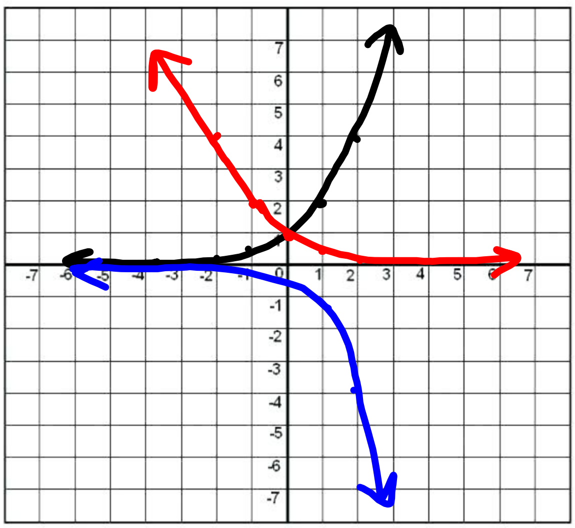


Graph the following functions on a calculator and sketch.

$f(x) = 2^x$

-2^x



$f(x) = \left(\frac{1}{2}\right)^x$

What did you notice about the graphs and their equations?

Is it a "reflection"? Why or why not? Not a reflection caused by –.

When b>1, the function represents **exponential** growth

When 0<b<1, the function represents **exponential** decay

$f(x) = 1^x$

Exponential Growth and Decay

$$f(t) = a(1 \pm r)^t$$

f(t) = value of the function after time (t)

a = initial value

r = interest rate (written in decimal form)

t = time (in years unless otherwise stated)

John researches a baseball card and find that it is currently worth \$3.25. However, it is supposed to increase in value 11% per year. $f(t) = a(1 \pm r)^t$ $(1+0.11)$

$0.11 = r$

a) Write an exponential equation to represent this situation

$f(t) = 3.25(1.11)^t$

b) How much will the card be worth in 10 years?

$= 3.25(1.11)^{10} = \$9.23$

c) Use your graphing calculator to determine in how many years will the card be worth \$26.

On federal income tax returns, self employed people can depreciate the value of business equipment. Suppose a computer valued at $2765 depreciates at a rate of 30% per year. $f(t) = a(1 \pm r)^t$

a) Write an exponential equation to model this situation

b) How much will this computer be worth in 5 years?

c) Use your graphing calculator to determine in how many years will the computer be worth $350.

The population of Orem in 1950 was 4,000 and was increasing at a rate of 2.6% per year.

a) Predict the population of Orem in 1975 and 2000.

b) Using your graphing calculator, predict when Orem's population will hit 200,000 people.

You try! ☺

As a birthday present you received a pair of track shoes signed by Mr. Myrup that is valued at $500 (ya know cause he's so awesome). Over time the value increases at a rate 5.5% per year.

a) Write an exponential equation to represent the situation.

b) How much will the shoes be worth after 7 years?

c) How long until they are worth $1000?

Compound Interest Formula

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

A(t) is the value after time (t)

P is the principal

r is the annual interest rate

n is the number of compounding periods per year

t is the time in years

Write an equation then find the final amount for each investment.

a. \$1000 at 8% compounded semiannually for 15 years

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

You Try!

b. \$1750 at 3.65% compounded daily for 10 years

Using a calculator, determine how many years it will take for the amount to reach \$4000.

Write an equation then find the final amount for each investment.

a) An investment of $1000 comounded monthly at a rate of 4.5%.

b) How much money is there after 5 years?

c) How long until the investment has tripled its value?

Investigate the growth of $1 investment that earns 100% annual interest (r=1) over 1 year as the number of compounding periods, n, increases. Do this with a group/partner.

Compounding schedule	n	$1\left(1+\frac{1}{n}\right)^n$	Value of A
annually	1		
semiannually	2		
quarterly	4		
monthly	12		
daily	365		
hourly	8760		
every minute	525600		

What does the value of A approach?

The value e is called the natural base

The exponential function with base e, $f(x)=e^x$, is called the natural exponential function.

$$e \approx 2.71828182827$$

what you need to know is $e \approx 2.7$

Evaluate $f(x)=e^x$ for

a. $x = 2$

b. $x = \frac{1}{2}$

c. $x = -1$

Many banks compound the interest on accounts daily or monthly. However, some banks compound interest continuously, or at every instant, by using the *continuous compounding formula.*

Continuous Compounding Formula

If *P* dollars are invested at an interest rate *r*, that is compounded continuously, then the amount, *A*, of the investment at time *t* is given by

$$A(t) = Pe^{rt}$$

A person invests $1550 in an account that earns 4% annual interest compounded continuously.

a. Write an equation to represent this situation

b. Using a calculator, find when the value of the investment reaches $2000.

An investment of $1000 earns an annual interest rate of 7.6%.

Compare the final amounts after 8 years for interest *compounded quarterly* and for interest *compounded continuously.*